Next-leading BFKL effects in forward-jet production at HERA

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We show that next-leading logarithmic (NLL) Balitsky-Fadin-Kuraev-Lipatov (BFKL) effects can be tested by the forward-jet cross sections recently measured at HERA. For $d\sigma/dx$, the NLL corrections are small which confirms the stability of the BFKL description. The triple differential cross section $d\sigma/dxdk_T^2dQ^2$ is sensitive to NLL effects and opens the way for an experimental test of the full BFKL theoretical framework at NLL accuracy.

1. Forward-jet production in deep inelastic scattering is a process in which a jet is detected at forward rapiditites in the direction of the proton. The virtuality of the intermediate photon Q^2 and the squared transverse momentum of the jet k_T^2 are two hard scales. When the total energy of the photon-proton collision W is assumed to be large, corresponding to a small value of the Bjorken variable x, forward-jet production is relevant [1] for testing the Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach [2].

Indeed in the small$-x$ regime, besides the large logarithms coming from the strong ordering between the proton scale and the forward-jet scale (which are resummed using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [3]), other large logarithms arise in the hard cross section itself, due to the ordering between the energy W^2 and the hard scales. These can be resummed using the BFKL equation, at leading (LL) and next-leading (NLL) logarithmic accuracy [2, 4]. By contrast, in fixed-order perturbative QCD calculations the hard cross section is computed at fixed order with respect to α_s, and the next-to-leading order (NLO) predictions fail to describe the data.

NLL corrections to the LL-BFKL kernel were found to feature spurious singularities. It has been realised that renormalisation-group improved NLL regularisations can solve this singularity problem [5, 6] and lead to consistent NLL-BFKL kernels. This, along with the success [7, 8] of the LL-BFKL approach in describing the $d\sigma/dx$ data, motivates the present phenomenological analysis of NLL-BFKL effects in forward-jet production.

We obtain that for $d\sigma/dx$ the NLL corrections are small, which results in a good description of the H1 data by NLL-BFKL predictions. We also show that the recently measured triple differential cross section $d\sigma/dxdk_T^2dQ^2$ [9] allows for a detailed study of the NLL-BFKL approach and of the QCD dynamics of forward jets. In particular, it has the potential to address the question of the remaining ambiguity corresponding to the dependence on the specific regularisation scheme of the NLL kernel.

The present study is a phenomenological analysis of the new forward-jet data using NLL-BFKL predictions related to the regularisation schemes. In Ref.[10], such a phenomenological investigation has been devoted to the proton structure function data, taking into account NLL-BFKL effects through an "effective kernel" (introduced in [6]) using three different schemes. A saddle-point approximation for hard enough scales is used in order to obtain a phenomenological description of NLL-BFKL effects. In the present study devoted to forward-jet production, we implement them in a similar way. This allows to study the NNL-BFKL framework even though the determination of the next-leading impact factors is still in progress [11].

2. We shall use the usual kinematic variables of deep inelastic scattering: $x = Q^2/(Q^2+W^2)$ and $y = Q^2/(xs)$ where $\sqrt{s}$ is the center-of-mass energy of the lepton-proton collision. In addition, x_J is the jet longitudinal momentum fraction with respect to the proton. The fully differential cross section for forward-jet production reads

$$\frac{d^4\sigma}{dxdQ^2dx_Jdk_T^2} = \frac{\alpha_{em}}{\pi x Q^2} \sum_{\lambda=L,T} f_\lambda(y)\, \frac{d\sigma_\lambda^{\gamma* p\to JX}}{dx_J dk_T^2} \qquad (1)$$

with $f_T(y) = 1 - y + y^2/2$, $f_L(y) = 1 - y$, and where $d\sigma_{T,L}^{\gamma* p\to JX}/dx_J dk_T^2$ is the cross section for forward-jet production in the collision of transversely (T) or longitudinally (L) polarized virtual photons with the proton.

In the following, we consider the high-energy regime $x \ll 1$ in which the rapidity interval $Y = \log(x_J/x)$ is assumed to be large. Following the phenomenological NLL-BFKL analysis of [10], one obtains for the forward-jet cross section:

$$\frac{d\sigma_{T,L}^{\gamma* p\to JX}}{dx_J dk_T^2} = \frac{\alpha_s(k_T^2)\alpha_s(Q^2)}{k_T^2 Q^2}\, f_{eff}(x_J, k_T^2) \int \frac{d\gamma}{2i\pi}\left(\frac{Q^2}{k_T^2}\right)^\gamma \phi_{T,L}^\gamma(\gamma)\, e^{\bar\alpha(k_T^2)\chi_{eff}\left(\gamma,\bar\alpha(k_T^2)\right)Y} \qquad (2)$$

with the running coupling constant given by

$$\bar\alpha(k^2) = \alpha_s(k^2)N_c/\pi = \left[b\log\left(k^2/\Lambda_{QCD}^2\right)\right]^{-1} \qquad (3)$$

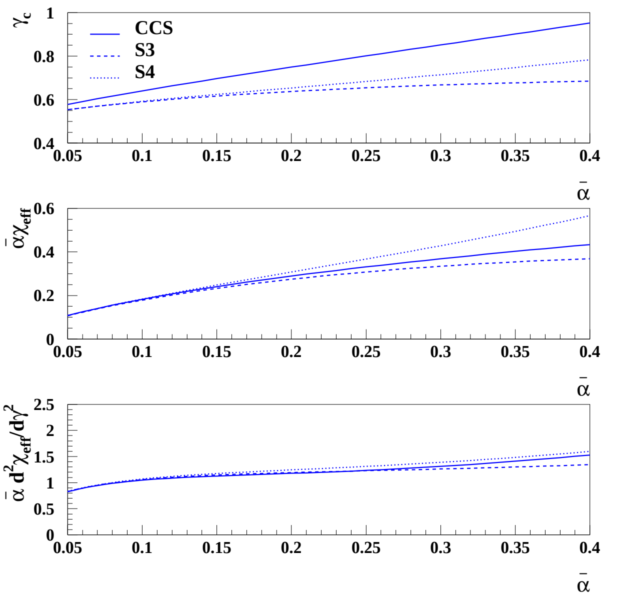

FIG. 1: γ_c, $\bar{\alpha}\chi_{eff}(\gamma_c,\bar{\alpha})$ and $\bar{\alpha}\chi''_{eff}(\gamma_c,\bar{\alpha})$, as functions of $\bar{\alpha}$ for the three BFKL resummation schemes CCS, S3 and S4.

where $b = 11/12 - N_f/6N_c$. We have chosen to consider $\bar{\alpha}(k_T^2)$ and tested the sensitivity of our results when using $\bar{\alpha}(Qk_T)$ or $\bar{\alpha}(Q^2)$ instead.

In formula (2), $f_{eff}(x_J, k_T^2) = g + (q+\bar{q})C_F/N_c$ is the effective parton density which resums the leading logarithms in $\log(k_T^2/\Lambda_{QCD}^2)$. g (resp. q, $\bar{q}$) is the gluon (resp. quark, antiquark) distribution function in the incident proton. Since the forward jet measurement involves large values of k_T and moderate values of x_J, formula (2) features the collinear factorization of f_{eff}, with k_T^2 chosen as the factorization scale.

The NLL-BFKL effects are phenomenologically taken into account by the effective kernel $\chi_{eff}(\gamma,\bar{\alpha})$. Indeed, the NLL-BFKL kernels $\chi_{NLL}(\gamma,\omega)$ provided by the regularisation procedure obey a *consistency condition* [5] which allows to reformulate the problem in terms of $\chi_{eff}(\gamma,\bar{\alpha})$. We shall consider the CCS scheme [6] and the S3 and S4 schemes [5] in which χ_{NLL} also depends explicitly on $\bar{\alpha}$. In each case, the effective kernel $\chi_{eff}(\gamma,\bar{\alpha})$ is obtained from the NLL kernel $\chi_{NLL}(\gamma,\omega)$ by solving the implicit equation:

$$\chi_{eff} = \chi_{NLL}(\gamma, \bar{\alpha}\ \chi_{eff})\ , \tag{4}$$

as a solution of the consistency condition.

The details of the approximation (2) are given in [10], with the only difference that the kernel considered for F_2 was naturally the asymmetric one $\chi_{NLL}(\gamma+\omega/2,\omega)$. In the forward-jet problem, the energy scale is considered to be given symmetrically between the hard probes by $\log(W^2/Qk_T)$ instead of $\log(W^2/Q^2) \simeq \log(1/x)$ as was the case for F_2. In other words, we do not perform the shift $\gamma \to \gamma + \omega/2$ used for F_2.

It is important to note that in formula (2), we use the leading-order (Mellin-transformed) impact factors given by

$$\begin{pmatrix} \phi_T^\gamma(\gamma) \\ \phi_L^\gamma(\gamma) \end{pmatrix} = \pi\alpha_{em} N_c^2 \sum_q e_q^2 \frac{1}{2\gamma^2} \begin{pmatrix} (1+\gamma)(2-\gamma) \\ 2\gamma(1-\gamma) \end{pmatrix} \frac{\Gamma^3(1+\gamma)\Gamma^3(1-\gamma)}{\Gamma(2-2\gamma)\Gamma(2+2\gamma)(3-2\gamma)} \tag{5}$$

as the full next-leading impact factors are not yet available. We point out that our phenomenological approach can be adapted to full NLL accuracy, once the next-leading impact factors available. For completion, we shall discuss the sensibility of our results with respect to modifications of $\phi_{T,L}^\gamma(\gamma)$.

Expressing the integral in (2) using a saddle-point approximation in γ, one finds for the theoretical forward-jet cross section

$$\frac{d\sigma_{T,L}^{\gamma^* p \to JX}}{dx_J dk_T^2} \simeq \frac{\alpha_s(k_T^2)\alpha_s(Q^2)}{k_T^2 Q^2} f_{eff}(x_J, k_T^2)\phi_{T,L}^\gamma(\gamma_c) \left(\frac{Q^2}{k_T^2}\right)^{\gamma_c} e^{\bar{\alpha}\chi_{eff}(\gamma_c,\bar{\alpha})Y} \frac{\exp\left(-\frac{\log^2(Q^2/k_T^2)}{2\bar{\alpha}\chi''_{eff}(\gamma_c,\bar{\alpha})\ Y}\right)}{\sqrt{2\pi\bar{\alpha}\chi''_{eff}(\gamma_c,\bar{\alpha})\ Y}}\ , \tag{6}$$

where $\chi''_{eff} = d^2\chi_{eff}/d\gamma^2$, and where the saddle point equation is $\chi'_{eff}(\gamma_c,\bar{\alpha}) = 0$. It is possible to extract the values of γ_c, $\bar{\alpha}\chi_{eff}(\gamma_c,\bar{\alpha})$ and $\bar{\alpha}\chi''_{eff}(\gamma_c,\bar{\alpha})$ after solving the implicit equation (4). They are given in Fig.1 for the different schemes, as functions of $\bar{\alpha}$.

The description of the forward-jet cross section is then almost parameter free; the value of $\bar{\alpha}$ is imposed by the renormalisation group equations and only the overall normalisation is unknown (only the knowledge of the next-leading impact factors will provide a full prediction). By comparison, the LL-BFKL formula is formally the same as (6), but with $\chi_{eff} \to \chi_{LL}(\gamma)$ ($\gamma_c \to 1/2$) and $\bar{\alpha}(k^2) \to \bar{\alpha} = const.$ where

$$\chi_{LL}(\gamma) \equiv 2\psi(1) - \psi(1-\gamma) - \psi(\gamma) \tag{7}$$

and $\psi(\gamma) = d\log\Gamma(\gamma)/d\gamma$. In the LL-BFKL case, this is a two-parameter formula (normalisation and $\bar{\alpha}$). The interesting property of our phenomenological approach is that formula (2) has formally the structure of the LL formula, but with only one free parameter and a NLL kernel. The delicate aspect of the problem comes from the scheme-dependent effective kernel χ_{eff}.

3. The NLL-BFKL formula for the fully differential forward-jet cross section is obtained from (1) and (6), with γ_c, $\bar{\alpha}\chi_{eff}(\gamma_c,\bar{\alpha})$ and $\bar{\alpha}\chi''_{eff}(\gamma_c,\bar{\alpha})$ plotted in Fig.1 for the three different schemes. To fix the normalisation (the only free parameter) and check the quality of the data description using the BFKL formalism, we start by fitting the $d\sigma/dx$ H1 data [9]. The choice of this data set corresponds to the kinematical domain where the BFKL formalism is expected to hold ($x \ll 1$ and $Q^2/k_T^2 \sim 1$).

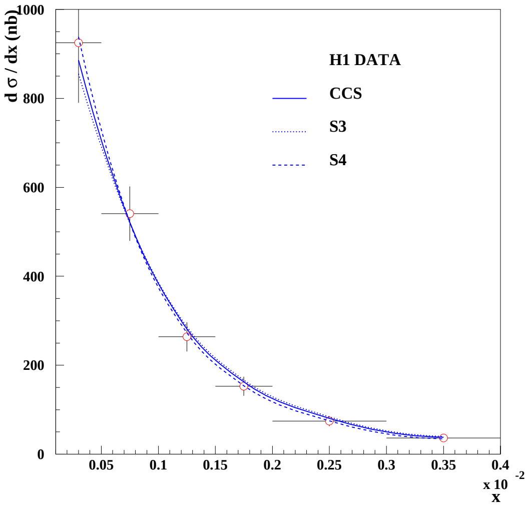

FIG. 2: The forward-jet cross section $d\sigma/dx$ measured by the H1 collaboration, compared with the three NLL-BFKL parametrizations S4, CCS and S3.

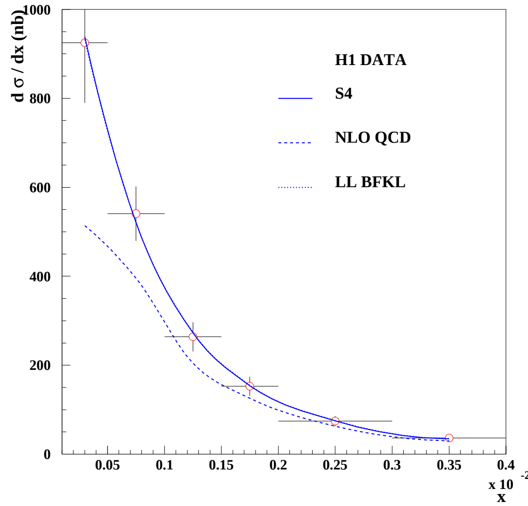

FIG. 3: The forward-jet cross section $d\sigma/dx$ measured by the H1 collaboration, compared with LL and NLL (S4) BFKL fits and with NLO QCD predictions.

The fitting procedure is the same as the one described in Ref.[8] (see Appendix A). The integrals over x_J, Q^2 and k_T^2 are performed numerically taking into account the kinematic cuts given in [9]. We performed fits on statistical errors only, systematics errors being strongly point-to-point correlated. In other words, we do as if the systematic errors were 100% correlated (which is close to reality). The fit results to the $d\sigma/dx$ H1 data are shown in Fig.2. The S4 fit can describe the data nicely ($\chi^2 = 5.6/5$ d.o.f.) whereas the S3 and CCS schemes show higher values of χ^2 ($\chi^2 = 45.9/5$ and $\chi^2 = 20.4/5$ respectively). Note that, when fitting using total errors (shown on the figures), all three schemes give values of χ^2 below 1. We also found that the results were unchanged when using $\bar{\alpha}(Qk_T)$ or $\bar{\alpha}(Q^2)$ in (6).

A comparison of the S4 fit with LL-BKFL results taken from [8] is shown in Fig.3. We first notice the tiny difference between the LL and NLL results (in fact, the corresponding curves are not distinguishable on the figure). This confirms that the data are consistent with the BFKL enhancement towards small values of x. Contrary to the proton structure function F_2, the forward-jet cross section $d\sigma/dx$ does not show large NLL-BFKL corrections. We also present in Fig.3 the fixed order QCD calculation based on the DGLAP evolution of parton densities. The next-to-leading order (NLO) prediction of forward-jet cross sections is obtained using the NLOJET++ generator [12]. CTEQ6.1M [13] parton densities were used, and the renormalization μ_r and the factorization scale μ_f were set equal to $\mu_r^2 = \mu_f^2 = Qk_t^{max}$, where k_t^{max} corresponds to the maximal transverse momentum of forward jets in the event. The NLO QCD predictions do not describe the data at small values of x, as they are below by a factor of order 2.

The fit parameters obtained with statistical error only are used in the following to make predictions for other observables. Namely, the relative normalisation between the different NLL BFKL calculations (CCS, S3 and S4) is used to make predictions for the triple differential cross section $d\sigma/dxdk_T^2dQ^2$. This is an interesting observable as it has been measured with 3 different k_T^2 and Q^2 cuts, yielding 9 different regions for the ratio $r = k_T^2/Q^2$. It was noticed in [8] that the LL-BFKL formalism leads to a good description of the data when r is close to 1 and deviates from the data when r is further away from 1, as effects due to the ordering between Q and k_T start to set in. NLO QCD predictions show the reverse trend.

The H1 data for $d\sigma/dxdk_T^2dQ^2$ are shown in Fig.4 and compared with the S4 prediction, the LL-BKFL results (taken directly from [8]) and NLO QCD predictions. It is quite remarkable that the NLL-BFKL calculation, which includes some ordering between Q and k_T, leads to a good description of the H1 data on the full range. As it was the case for $d\sigma/dx$, the difference between the LL and NLL results is small when $r \sim 1$. By contrast when r differs from 1, the difference is significant, and the observable $d\sigma/dxdk_T^2dQ^2$ is quite sensitive to NLL-BFKL effects. When using $\bar{\alpha}(Qk_T)$ or $\bar{\alpha}(Q^2)$ in (6), the results shown in Fig.4 differ by a small amount, at most 10%.

In fact, the triple differential cross section has the potential to resolve the scheme ambiguity. For instance, the predictions of the other schemes CCS and S3 do not compare with the data as well as the predictions of the S4 scheme. In addition, contrary to the S4 predictions, we observed that the shape of the CCS and S3 curves is quite sensitive to the variation of the impact factors [14]: as a first try to test the stability of our results with respect to the impact factors, we have moved the argu-

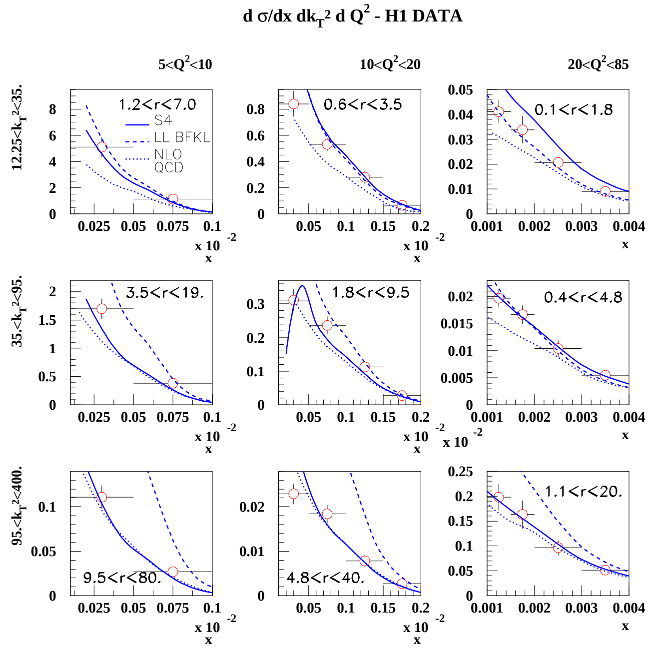

FIG. 4: The forward-jet cross section $d\sigma/dxdk_T^2dQ^2$ measured by the H1 collaboration, compared with LL and NLL (S4) BFKL fits and with NLO QCD predictions.

ment of $\phi_{T,L}^{\gamma}$ from 1/2 to γ_c. This indicates that, when the next-leading impact factors will be known, the predictions between different schemes could differ significantly, allowing for a strong test of NLL-BFKL predictions.

4. Let us summarize our main results. For the cross section $d\sigma/dx$, measured in the kinematical regime $Q^2/k_T^2 \sim 1$, the difference between the LL-BFKL and NLL-BFKL descriptions is very small (see Fig.3). This confirms the validity of the BFKL description of [7, 8] previously obtained with LL accuracy. In the case of the triple differential cross section $d\sigma/dxdk_T^2dQ^2$, the same conclusion holds when $r = k_T^2/Q^2 \sim 1$. In addition when r differs from 1, the NLL-BFKL description is quite different from the LL-BFKL one, as it is closer to the NLO QCD calculation as expected. As a result, the best overall descrition of the data for $d\sigma/dxdk_T^2dQ^2$ is obtain with the NLL-BFKL formalism and the S4 scheme is favored. We need the complete knowledge of the next-leading impact factors before drawing final conclusions, but our analysis strongly suggests that the data show the BFKL enhancement at small values of x. This is of great interest in view of the LHC, where similar QCD dynamics will be tested with Mueller-Navelet jets [15, 16].

We would like to thank Laurent Schoeffel and Sebastian Sapeta for providing their numerical evaluations of the effective kernel.

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